Table 3. Conductance ratio $U_{(4)}/U_{(2)}$ for various $H/D$

| $Re_{(2)}$ | $H/D$ 4 | 8 | 12 |
|---|---|---|---|
| 600 | 0.419 | 0.437 | 0.420 |
| 1000 | 0.409 | 0.421 | 0.416 |
| 1800 | 0.394 | 0.405 | 0.405 |
| 3000 | 0.384 | 0.389 | 0.401 |
| 5000 | 0.375 | 0.375 | 0.393 |

to 14% for $H/D = 8$. Further study of Table 3 shows that the conductance ratios for all three $H/D$ cases lie within 14% of each other. Table 3 also shows that the $U$ values for $S/D = 2$ are substantially greater than those for $S/D = 4$. This definitive finding will be elaborated shortly.

Although the results in Table 3 have a larger spread than those in Table 2, neither spread is very great considering the order of magnitude range of the Reynolds number. For a compact presentation, average values may be obtained for $U_4/U_{12}$, $U_8/U_{12}$ and $U_{(4)}/U_{(2)}$. The end result of such an averaging is

$$U_4/U_{12} = 0.434, \quad U_8/U_{12} = 0.733 \tag{12}$$

for $S/D = 2$ and 4, and

$$U_{(4)}/U_{(2)} = 0.403 \tag{13}$$

for $H/D = 4$, 8 and 12.

## CONCLUSIONS FROM THE PERFORMANCE STUDY

The foregoing performance calculations, utilizing a well defined set of constraints and objectives, and based on data from experiments described in [1], has yielded an extremely compact set of results which lend themselves to use in design applications. According to Tables 2 and 3 or from their condensed forms, equations (12) and (13), it follows that the lowest overall resistance corresponding to a fixed pumping power is obtained for the longest possible fins and the smallest possible inter-fin spacings.

These findings should, however, not be employed *verbatim*. In particular, the fin efficiency, assumed to be unity in the present analysis, diminishes with increasing fin length. The accounting of this factor, which depends upon the specifics of the fin material, would arrest the aforementioned trend whereby the overall resistance of the array decreases monotonically with fin length. A fin efficiency $\eta$ less than unity can be incorporated into the analysis by introducing $\eta_i$ as a multiplying factor of $Nu_i$ in equation (11).

A second limiting factor is the array pressure drop, which is expected to grow rapidly as $S/D$ approaches one. This behavior would reverse the trend of increasing $U$ values with decreasing $S/D$.

Thus, the present results serve as definitive guidelines to design, but they should not be employed capriciously outside the range of the calculations on which they are based.

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# THERMAL DESIGN OF A HEAT EXCHANGER EMPLOYING LAMINAR FLOW OF PARTICLE SUSPENSIONS

AVTAR S. AHUJA
Case Western Reserve University, Wearn Research Building,
2074 Abington Road, Cleveland, OH 44106, U.S.A.

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## INTRODUCTION

IT IS WELL known that transport properties (viscosity, thermal conductivity and mass diffusion coefficient) of single-phase fluids are independent of the state of motion in the laminar regime. It has been shown experimentally that the thermal conductivity or diffusivity is augmented in the laminar flow of suspensions of polystyrene spheres [1, 2]. The augmentation, seen to be as much as 200% [2], is a function of particle radius $a$ and volume fraction $\phi$, heated length $L$ and i.d. $D$ of tube, shear rate or particle angular velocity $\omega$, and transport properties (kinematic viscosity $\nu$ and thermal diffusivity $\alpha$) of suspending fluid. By constructing dimensionless groups from these variables [3] or by following a systematic dimensional analysis [4], augmentation of the thermal diffusivity of a flowing suspension has been correlated [3] on the basis of groups such as $\omega a^2/\nu$, $\omega a^2/\alpha$, $D/a$, $L/a$ and $\phi$, and the effects of particle interactions on heat transport in flowing suspensions have been postulated. By reducing the data on a common set of coordinates constructed out of these groups of variables, heat and gas transports in flowing suspensions have been shown to be analogous [5, 6].

In this paper, the heat transfer data reported earlier [2] have been interpreted for the thermal design of a heat exchanger which employs suspensions in laminar motion. Remarkable enhancements of Nusselt number and heat exchanger effectiveness have been demonstrated as polystyrene spheres are added to a single-phase liquid. Thermal design of a blood heat exchanger (blood may be treated as a suspension of red cells in plasma) has been presented elsewhere [7].

There is a great deal of work available in literature [8–10] covering a variety of methods for enhancing heat transfer. However, to the present author's knowledge, the method of augmenting heat transfer delineated herein is not avilable in the literature.

## THEORY

Suspension or other test liquid is heated in a countercurrent heat exchanger. The theory involved in the determination of the overall heat transfer coefficient $h$ and the Nusselt number $Nu$ has been described elsewhere [1, 2, 7] and will not be repeated here. For computations of Nusselt numbers of this paper thermal conductivity values of stationary suspensions [11] have been employed.

*Power expenditure.* Power expended is

$$W = V\,\Delta P \tag{1}$$

where $V$, the volume flow rate, is $\pi D^2 u/4$ and $u$ is the mean velocity. The pressure drop [12] is

$$\Delta P = \frac{1}{2} f \frac{L}{D} \rho u^2 \tag{2}$$

where $\rho$ is the density and $f$, the friction factor, for a laminar flow is a well known relation [12]:

$$f = 64/Re, \tag{3}$$

and the Reynolds number of tube flow is $Re = uD/\nu$. It has been shown experimentally [2] that equation (3) holds for polystyrene suspensions of volume fractions up to 11%. By employing equations (1)–(3), the expression for power expended in laminar flow becomes

$$W = 8\pi\rho \left(\frac{L}{D}\right)^3 \frac{1}{D} Gz^2\, \nu\alpha^2 \tag{4}$$

where the Graetz number $Gz = Re\ Pr\ D/L$ and the Prandtl number $Pr = \nu/\alpha$. For a given tube ($L$ and $D$) and Graetz number $Gz$, power expenditure is proportional to $\eta\alpha^2$, and the augmentation in power expenditure is given by

$$\frac{W_s}{W} - 1 = \frac{\eta_s}{\eta}\left(\frac{\alpha_s}{\alpha}\right)^2 - 1 \tag{5}$$

where subscript s denotes suspension. Measured values of power expended in propelling suspensions [2], computed from equation (1), are plotted in Fig. 1 against Graetz number. Power expenditure for 4.64% suspension prepared in aqueous NaCl is not more than 10% over and above that for propelling the suspending liquid (Fig. 1). This value is in close agreement with the value (8%) computed from equation (5).

*Heat exchanger effectiveness.* The expression for heat exchanger effectiveness for countercurrent flow [12] can be simplified to

$$E = (t_{se} - t_{si})/(t_{wi} - t_{si})$$
$$= 1 - \exp\left[hA/(mc)_s\right] \text{ for } (mc)_s/(mc)_w = 0 \tag{6}$$

where $t$ is the temperature, $h$ is the average overall heat transfer coefficient, $m$ is the mass flow rate ($= \rho V$), $c$ is the specific heat, and subscripts s, w, i and e respectively denote suspension, water, inlet and exit. The quantity $hA/(mc)_s$ is termed the number of heat transfer units (NTU). Since in these heat transfer experiments the ratio $(mc)_s/(mc)_w$ (=0.0007–0.0006) is a very small value, equation (6) is applicable.

## MATERIALS AND METHODS

The experimental procedure has been detailed earlier [1, 2, 7] and will not be repeated here. It should be pointed out that as the suspension flowed, temperature of the tube wall varied, and the experimental apparatus was calibrated with tap water [2, 7]. Specifications of tubes and suspensions have been provided in captions of figures. The suspending liquids were selected in such a way as to render the suspensions neutrally buoyant. The inlet temperature of suspensions $t_{si}$ varied from 1–14°C and the outlet temperature $t_{se}$ varied from 28–44°C. The accuracy of the measurements was to within 10%.

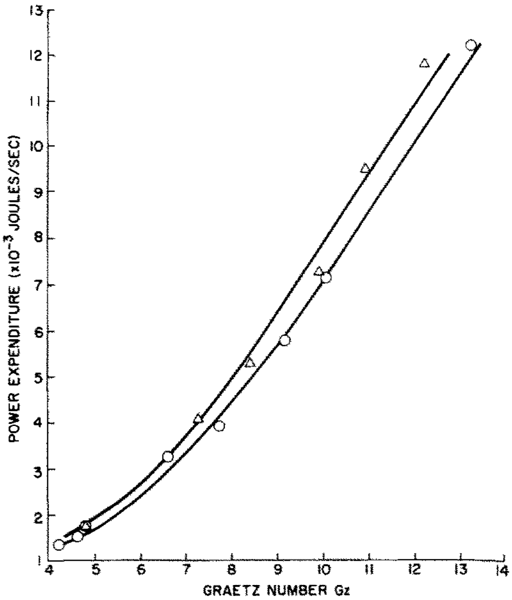


FIG. 1. Plots of power expenditure versus Graetz number for the laminar flow of suspending liquids and suspensions in a tube of inside diameter $D$ = 1 mm and length $L$ = 55.2 cm. ○, suspending liquid: 5.5 weight percent aqueous NaCl, $Re$ = 382–1204; △, suspension of polystyrene spheres of diameter $2a$ = 100 μm, volume fraction $\phi$ = 4.64% and prepared in 5.2 weight percent aqueous NaCl, $Re$ = 379–960.

## RESULTS AND DISCUSSION

Although a large amount of data was acquired, in the interest of space, selected data are presented in the following illustrations.

In the following discussion, Nusselt number $Nu$ is defined as $h_1 D/k$, where $h_1$ is the average (inside) heat transfer coefficient of the test liquid and $k$ is the thermal conductivity. The overall heat transfer coefficient $h$ (Fig. 2) and the Nusselt number (Figs. 3 and 4) acquire greater values as the particle volume fraction or the Graetz number is increased. The augmentation trends in Figs. 2–4 can be explained succinctly in terms of a single parameter, namely $\omega a^2/\nu$ as indicated in the introduction. It should be emphasized that, although particle rotation in shear flow has been observed and

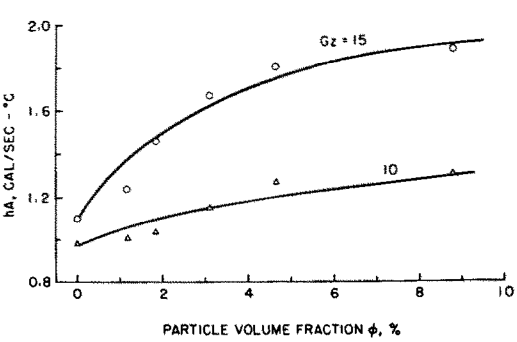


FIG. 2. Plot $hA$ vs the particle volume fraction $\phi$ for the laminar flow of test liquids. The average overall heat-transfer coefficient is $h$ and $A$ is the area for heat exchange. Suspensions of 100 μm dia. polystyrene spheres and 5.2 weight percent aqueous NaCl flowed in a tube of inside diameter $D$ = 1 mm and heated length $L$ = 55 cm. ○, $Gz$ = 15, and △, $Gz$ = 10.

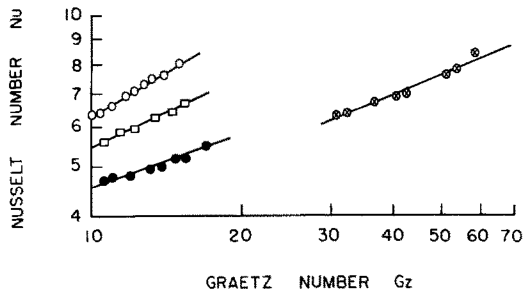


FIG. 3. Plots of $Nu$ vs $Gz$ for the laminar flow of polystyrene suspensions of volume fraction $\phi = 4.64\%$. ○, particle diameter $2a = 100\,\mu m$, suspending liquid: 5.2 weight percent aqueous NaCl, tube $D = 1$ mm and $L = 55$ cm, $Re = 911$–$1372$; □ Particle diameter $2a = 100\,\mu m$, suspending liquid: 20 weight percent aqueous glycerine, tube inside diameter $D = 1$ mm and heated length $L = 55$ cm, $Re = 560$–$805$; ● Particle diameter $2a = 50\,\mu m$, suspending liquid: 5.2 weight percent aqueous NaCl, tube $D = 1$ mm and $L = 55$ cm, $Re = 968$–$1529$; and ⊗ particle diameter $2a = 100\,\mu m$, suspending liquid: 5.2 weight percent aqueous NaCl, tube $D = 2$ mm and $L = 40$ cm, $Re = 1004$–$1906$.

documented in published literature, the influence of groups such as $\omega a^2/\nu$ on heat transfer is deduced from the similarity arguments [3] and dimensional analysis [4]. As the value of this parameter exceeds unity, the churning of the fluid around the rotating particle is established and the heat transfer is augmented. The fluid churning can be altered by varying the angular velocity $\omega$ or the shear rate, the particle radius $a$, and the suspending fluid kinematic viscosity $\nu$.

Larger values of $h$ (figure omitted) and of $Nu$ (Fig. 3) are obtained when, given other factors, suspensions are prepared in aqueous sodium chloride than in aqueous glycerine. Since the kinematic viscosity $\nu$ of aqueous glycerine is 55% larger than that of aqueous sodium chloride [2], for constant $\omega$ and $a$, the value of $\omega a^2/\nu$ or the churning effect reduces when particles are suspended in aqueous glycerine in comparison to that in aqueous NaCl.

When only the particle size is varied, a $100\,\mu m$ dia. particle yields larger values of $h$ (figure omitted) and of $Nu$ (Fig. 3) than the values obtained with a $50\,\mu m$ dia. particle. Once again for constant values of $\omega$ and $\nu$ reduction in $a$ is tantamount to reduction in the value of $\omega a^2/\nu$ or in the churning of the fluid surrounding the particle.

When only the tube dimensions are altered, 1 mm dia. tube yields larger values of $h$ (figure omitted) and of $Nu$ (Fig. 3) than the values obtained in a 2 mm dia. tube. This can be seen by extrapolation in Fig. 3. In contrast, one may recall that for single phase fluids $Nu$ remains independent of tube dimensions [12]. For a given value of the Graetz number $Gz$, reduction in $h$ or $Nu$ in a 2 mm dia. tube may be explained once again in terms of the parameter $\omega a^2/\nu$. For constant values of $a$ and $\nu$ and at a given value of $Gz$, particle angular velocity $\omega$ in a 2 mm tube is very much reduced as compared to the value in a 1 mm tube and the churning effect in a 2 mm tube is less than that in a 1 mm tube.

The overall heat transfer coefficient $h$ plotted in Fig. 2

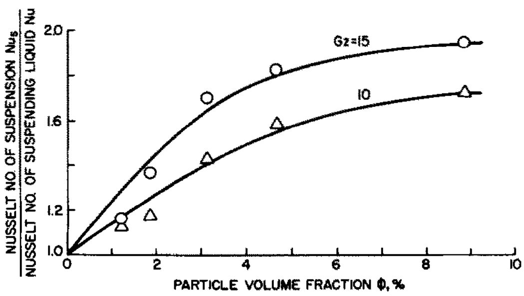


FIG. 4. Plots of $Nu_s/Nu$ vs particle volume fraction for the laminar flow of test liquids. Suspensions of $100\,\mu m$ dia. particles and 5.2 weight percent aqueous NaCl flowed in a tube of inside diameter $D = 1$ mm and heated length $L = 55$ cm. ○, $Gz = 15$; and △, $Gz = 10$.

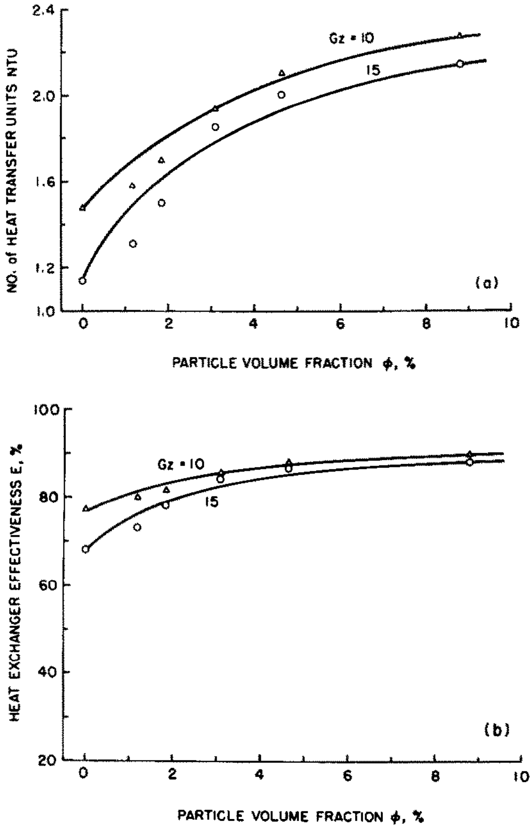


FIG. 5. Plots of (a) NTU and (b) heat exchanger effectiveness $E$ vs particle volume fraction $\phi$ for the laminar flow of test liquids. Suspensions of polystyrene spheres of diameter $2a = 100\,\mu m$ and 5.2 weight percent aqueous NaCl flowed in a tube of inside diameter $D = 1$ mm and heated length $L = 55$ cm. ○, $Gz = 15$; and △, $Gz = 10$.

Table 1. Percentage augmentation in various heat transfer quantities for $Gz = 15$ and percentage increase in work expended in the laminar flow of polystyrene suspensions

| | Particle volume fraction, $\phi$ (%) | | | |
|---|---|---|---|---|
| Augmented quantities* | 1.86 | 3.09 | 4.64 | 8.84 |
| Nusselt number | 36 | 70 | 82 | 95 |
| Heat exchanger effectiveness | 15 | 21 | 25 | 29 |
| Work expended | 4 | 6 | 8 | 20 |

Tube $D = 1$ mm and $L = 55$ cm, particle diameter $= 100\,\mu m$, suspending liquid is 5.2 weight percent aqueous NaCl. Temperature = 35°C.

* Augmentation in Nusselt number is defined as $(Nu_s/Nu - 1)$. Augmentation in heat exchanger effectiveness and an increase in work expended are defined similarly.

against the particle volume fraction $\phi$ for $Gz$ of 10 and 15 increases sharply up to volume fraction of 5% beyond which the increase is very slow. For a $Gz$ of 15 and a suspension of volume fraction of 5% the overall heat transfer coefficient is 1.6 times that for the aqueous NaCl.

The ratio of Nusselt number of flowing suspensions, $Nu_s$, to that of aqueous NaCl, $Nu$, is plotted against the particle volume fraction $\phi$ in Fig. 4. It increases linearly with $\phi$ sharply up to 5% beyond which the increase is very slow. Number of heat transfer units, NTU, is plotted against $\phi$ with $Gz$ as parameter in Fig. 5(a). At a given $Gz$, greater particle volume fractions yield larger values of NTU which means, as is apparent in equation (6), larger values of heat exchanger effectiveness. This conclusion is shown more clearly in Fig. 5(b).

Augmentation values of Nusselt number from Fig. 4 and of heat exchanger effectiveness from Fig. 5(b) both at $Gz$ of 15, are included in Table 1. By employing the values of the properties assembled elsewhere [2], the increases in power expenditure for propelling suspensions (computed from equation (5)) are also included in Table 1. It is seen that the Nusselt number and the heat exchanger effectiveness are enhanced significantly while the increase in the power expenditure is much lower.

## SUMMARY

By a series of experiments, the existence of augmentation of heat transfer in laminar flow of particle suspensions has been demonstrated. The origin of this augmentation resides in the shear induced particle rotations and the concomitant churning of the embedding fluid. Effects of particle diameter and volume fraction, tube dimensions, suspending fluid diffusivities and shear rate on heat transfer have been explored and found to be appreciable. The heat transfer data have been presented for the purpose of the thermal design of a heat exchanger. Relative to heat transfer in pure suspending liquid, Nusselt number is augmented by 82% for a 100 $\mu$m dia. particle suspension of volume fraction of 4.64% at Graetz number of 15. The heat exchanger effectiveness is enhanced by 25%, while the energy expenditure for propelling the suspension is increased by only 8%.

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# RADIATION CONFIGURATION FACTORS FOR OBLIQUELY ORIENTED FINITE LENGTH CIRCULAR CYLINDERS

ALIASGHAR AMERI and JAMES D. FELSKE
Department of Mechanical and Aerospace Engineering, State University of New York at Buffalo, Amherst, NY 14260, U.S.A.

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## NOMENCLATURE

- $C$, center-to-center spacing of cylinders in the parallel orientation [m];
- $F$, configuration factor;
- $L$, length of cylinder [m];
- $R$, cylinder radius [m].

Greek symbols

- $\theta, \phi$, cylinder rotations defined in Fig. 1 [deg].

Subscripts

- max, pertaining to the maximum value of the configuration factor for a ninety degree orientation;
- 0, parallel orientation, finite length cylinders;
- $0_\infty$, parallel orientation, infinitely long cylinders;
- $\theta$, cylinder rotation in the theta direction;
- $\phi$, cylinder rotation in the phi direction.

Superscript

- $'$, end-point rotation.